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Deconstructing non-abelian gauge theories at one loop

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Abstract: Deconstruction of 5D Yang-Mills gauge theories is studied in next-to-leading order accuracy. We calculate one-loop corrections to the mass spectrum of the non-linear gauged σ-model, which is the low energy effective theory of the deconstructed theory. Renormalization is carried out following the standard procedure of effective field theories. The relation between the radius of the compactified fifth dimension and the symmetry breaking scale of the non-linear σ-model is modified by radiative corrections. We demonstrate that one can match the low lying spectrum of the gauge boson masses of the effective 4D gauged non-linear σ-model to the Kaluza-Klein modes of the 5D theory at one-loop accuracy.

Keywords: Field Theories in Higher Dimensions, Sigma Models, Chiral Lagrangians.

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Contents

1. Introduction

In the last few years the idea that at short distances the fundamental interactions may be best described by field theories with one time and more than three space dimensions had an enormous impact on particle physics model building [1, 2]. It has been used in attempts to solve the gauge hierarchy problem, to cure shortcomings of GUT models, to build models for the flavor structure and to construct new types of gauge and supersymmetry breaking mechanisms just to mention a few of the numerous applications (see [3] and references therein). In the simplest physical picture the additional space dimensions are compact[1] and at distances large compared to the size of these dimensions the physics appears to be four dimensional. At energies higher than the inverse radius of the compact dimensions the low lying Kaluza-Klein states can be excited. At even higher energies the full nature of higher dimensional physics will be revealed. The difficulty is, however, that higher dimensional theories are not renormalizable and therefore ill defined as quantum field theories. They can only be used as effective field theories (EFTs) defined with a cutoff and limited range of validity.[2] At energies close to the cutoff they become typically strongly coupled and their predictions will depend strongly on the cutoff scheme. These difficulties may be resolved by an ultra violet (UV) completion which respects all the important symmetries of the quantum field theory (Lorentz invariance, gauge invariance, SUSY, etc). For example, 5D supersymmetric Yang-Mills theories could be embedded in various ways into string theory or M-theory [8]. However, a pure QFT embedding is also possible [9, 10], in particular extra

[1] Infinitely large extra dimensions with warped geometries have also been suggested [4].

[2] Field theories in five or six dimensions may exist around a non-gaussian ultra-violet fixed point [5, 6, 7].

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dimensions can be generated dynamically from anomaly free asymptotically free gauge theories. The symmetries and particle content of the theory are represented by "moose" diagrams in "theory space" with sites and links. Locality and particle propagation in the extra dimensions are maintained due to the nearest neighbor interactions. In simple models at low energies the link fields are non-linear σ-model Nambu-Goldstone boson fields, they are mostly higgsed to give massive gauge bosons with a spectrum which may match the Kaluza-Klein spectrum of compactified extra dimensions. Therefore within 4D models one can reproduce an apparently higher dimensional mechanism. In the simplest cases the "moose" diagrams lead to discretized actions of extra dimensional gauge theories where only the extra dimensions are latticized. Deconstruction by means of an EFT can be viewed as a gauge-invariant regulator of the 5D theory,[3] where the lattice spacing corresponds to the cutoff of the higher dimensional theory. However unlike the 5D EFT, the deconstructed theory can be rendered manifestly renormalizable by choosing a UV-completion for the non-linear σ-model. Such a completion is for example provided by introducing a QCD-like strongly interacting sector or by a linear σ-model with a heavy Higgs boson. Models related to deconstruction have been proven to be powerful also for physics beyond the standard model with a few sites and without any higher dimensional interpretation (little Higgs models) [12].

The mechanism of dimensional deconstruction has been demonstrated at tree-level by matching the low lying spectra and parameters of the 4D and higher dimensional theory. The correspondence has been established precisely and it has been shown that this also holds for supersymmetric theories [13, 14], even in the non-perturbative regime [15]. It is of interest to investigate this correspondence also beyond tree level. Corrections in the mass spectrum may have important phenomenological consequences for the higher dimensional gauge models. In some of these models, one-loop corrections may control the electroweak symmetry breaking. In recent calculations of the one-loop corrections of the gauge boson masses it was found that the corrections have a well defined finite part which vanishes in the limit of a large compactification radius [16]–[19]. The divergent contributions, however, are the same as in the uncompactified theory. In the case of U(1) gauge theory using the linearized σ-model [20] for deconstruction, these results have recently been reproduced.

In this paper we calculate one-loop corrections in the low energy EFT, which is described by a 4D non-linear σ-model with non-abelian gauge symmetry, and perform renormalization by adding counter terms in chiral perturbation theory. To recover the result of the 5D compactified gauge theory one should define the compactification radius. We show how the formula for the lattice spacing obtained by the tree level matching is modified by radiative corrections. The one-loop corrections obtained in terms of renormalized quantities of the 4D theory can then be reinterpreted in terms of 5D gauge coupling and compactification radius.

In section 2 we review the 4D SU(k) gauged non-linear σ-model together with some next-to-leading order terms in the low energy expansion of Goldstone boson interactions. In section 3 we present our results for the gauge and the pseudo Goldstone boson mass

[3]An overview of other possible regularizations of higher dimensional field theories can be found in [11].

and discuss the next-to-leading order matching to the 5D gauge theory results. Finally the appendices are devoted to some technical details on propagators, the construction of the $\mathcal{O}(p^4)$ lagrangian and on the individual diagrammatic contributions.

2. Effective field theory

A 5D SU(k) gauge theory with one dimension compactified on S_1 can be deconstructed using a 4D EFT that approximates the spectrum of the first N Kaluza-Klein modes [9, 10, 12]. The effective lagrangian is obtained by using a non-linear realization of the gauge symmetry, employing well-known techniques from chiral perturbation theory for low-energy QCD [21, 22] and strongly interacting electroweak symmetry breaking sectors [23, 24, 25]. The effective lagrangian is organized in a power series in small momentum[4] $p \ll \Lambda \sim 4\pi v$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots , \qquad \text{where} \qquad \mathcal{L}_2 = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{g.f.}} + \mathcal{L}_{\text{gh}} . \tag{2.1}$$

The kinetic terms are

$$\mathcal{L}_{\text{kin}} = \sum_{p=1}^{N} \left\{ v^2 \,\text{Tr}\left[(D_\mu U_p)^\dagger D^\mu U_p \right] - \frac{1}{2} \,\text{Tr}\left[F^p_{\mu\nu} F^{\mu\nu}_p \right] \right\} , \tag{2.2}$$

where the covariant derivative is defined by[5]

$$D_\mu U_p = \partial_\mu U_p - ig A^p_\mu U_p + ig U_p A^{p+1}_\mu , \qquad A^p_\mu = A^{p,a}_\mu T^a . \tag{2.3}$$

The unitary matrices U_p are given in terms of the pseudo Goldstone bosons

$$U_p = \mathrm{e}^{iG_p/v} , \qquad G_p = G^a_p T^a , \tag{2.4}$$

and transform linearly under gauge transformations

$$U_p \to V_p U_p V^\dagger_{p+1} , \qquad A^p_\mu \to V_p A^p_\mu V^\dagger_p + \frac{i}{g} V_p \partial_\mu V^\dagger_p . \tag{2.5}$$

The model can conveniently be described by the moose diagram in figure 1. We will assume throughout that N is even. The lagrangian $\mathcal{L}_{\text{kin}}$ is a gauged non-linear σ-model with N gauge groups, with equal gauge coupling g, provided we impose discrete translational invariance around the circle, $p \to p+1$. This model can also be viewed as a 5D pure-gauge theory formulated on $\mathbb{R}_4 \times S_1$, where the compactified dimension is latticized [26]. Note that the gauge fields A^μ_p live on the sites, whereas the σ-model fields U_p are associated with the link between two adjacent lattice sites. The correspondence is established by identifying the Goldstone boson field with the fifth component of the higher dimensional gauge field. At tree-level the identification is as

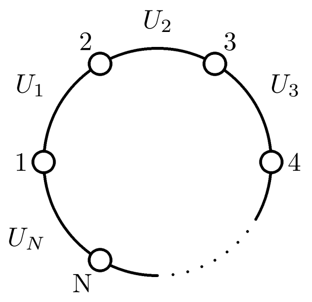

Figure 1: Moose diagram.

[4] We will discuss the relation between the expansion in powers of momentum p and in powers of the coupling constant g below.

[5] We use the following conventions for the generators $T^a\,(a = 1, \ldots, k^2-1)$ of SU(k): $\left[T^a, T^b\right] = i f^{abc} T^c$, Tr $\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$, as well as $f^{acd} f^{bcd} = C_2(G)\delta^{ab}$.

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follows (note that this relation receives corrections at the loop-level)

$$\frac{G_p}{v} = -agA_5^p\,, \qquad a^2 = \frac{1}{g^2v^2}\,, \qquad a = \frac{2\pi R}{N}\,. \tag{2.6}$$

Here a denotes the lattice spacing. The dimensionful gauge coupling of the 5D theory is given by $g_5^2 = ag^2$. From a physical point of view a is related to the cutoff of the 5D EFT ($\Lambda_{5\mathrm{D}} \sim a^{-1}$). Since for this theory to be valid it is necessary that $g_5^2\Lambda_{5\mathrm{D}} < 1$, it is automatically ensured that in the deconstructed theory one is in the perturbative regime.

By choosing an R_ξ-type gauge-fixing term

$$\mathcal{L}_{\text{g.f.}} = -\frac{1}{\xi}\sum_p \operatorname{Tr}\left[\partial_\mu A_p^\mu + \xi gv(G_p - G_{p-1})\right]^2 \tag{2.7}$$

we can remove the kinetic mixing term between the gauge and the Goldstone boson fields. The corresponding Faddeev-Popov ghost lagrangian can then be constructed in the usual way, although some care has to be taken due to the non-linear realization of the gauge symmetry. Following [27], it can be written in closed form as follows (see also [28])

$$\begin{aligned}\mathcal{L}_{\text{gh}} = \sum_p \Bigg[&\partial_\mu \bar{c}_p^a\,\partial^\mu c_p^a + gf^{abc}\partial_\mu\bar{c}_p^a A_p^{\mu,b}c_p^c - \\ &- \xi g^2v^2(\bar{c}_{p+1}^a - \bar{c}_p^a)\left(E^{-1}\left(\frac{\overline{G}_p}{v}\right)^{ab} c_{p+1}^b - E^{-1}\left(-\frac{\overline{G}_p}{v}\right)^{ab} c_p^b\right)\Bigg]\end{aligned} \tag{2.8}$$

where the matrix valued function $E(y)$ has the form

$$E^{-1}(y) = \frac{y}{1-e^{-y}} = 1 + \frac{y}{2} + \frac{y^2}{12} + \mathcal{O}\left(y^4\right) \qquad \text{and} \qquad \overline{G}_p^{ab} = f^{abc}G_p^c\,. \tag{2.9}$$

Expanding up to fourth order in the fields, one obtains explicitly

$$\begin{aligned}\mathcal{L}_{\text{gh}} = \sum_p \Bigg[&\partial_\mu\bar{c}_p^a\partial^\mu c_p^a + gf^{abc}\partial_\mu\bar{c}_p^a A_p^{\mu,b}c_p^c - \xi g^2v^2\left(\bar{c}_{p+1}^a - \bar{c}_p^a\right)\left(c_{p+1}^a - c_p^a\right) + \\ &+ \frac{\xi g^2 v}{2} f^{abc}\left(\bar{c}_{p+1}^a - \bar{c}_p^a\right)G_p^b\left(c_{p+1}^c + c_p^c\right) - \\ &- \frac{\xi g^2}{12} f^{abc}f^{cde}\left(\bar{c}_{p+1}^a - \bar{c}_p^a\right)G_p^bG_p^d\left(c_{p+1}^e - c_p^e\right) + (\text{higher terms})\Bigg]\,.\end{aligned} \tag{2.10}$$

Note that there appear new, quartic interaction vertices between Goldstone bosons and ghosts, compared to the case where the symmetry is linearly realized.

To diagonalize the kinetic lagrangian one expands the field in terms of Fourier-modes (see appendix A). The mass spectrum is given by

$$m_n^2 = 4g^2v^2\sin^2\frac{n\pi}{N} \tag{2.11}$$

for A_μ and by ξm_n^2 for the Goldstone boson. For large N, which means $R^{-1} \ll \Lambda_{5\mathrm{D}}$, the

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masses of the lowest lying modes become approximately independent of N and one gets an identical spectrum as in 5D Kaluza-Klein theories.

$$m_n^2 \simeq \frac{n^2}{R^2}\,, \qquad \text{for } n \ll N\,. \tag{2.12}$$

Although the non-linear σ-model lagrangian $\mathcal{L}_2$ in eq. (2.2) by itself is non-renormalizable, it is possible to remove all divergences in the EFT order by order in the low-energy expansion, since only a finite number of terms appear up to a given order in p. For instance, at $\mathcal{O}(p^2)$ there are only tree-level contributions from $\mathcal{L}_2$, whereas at $\mathcal{O}(p^4)$ one has to consider tree-graphs from $\mathcal{L}_4$ and one-loop graphs with vertices from $\mathcal{L}_2$. The lagrangian $\mathcal{L}_4 = \sum_i c_i \mathcal{O}_i$ then contains higher order operators $\mathcal{O}_i$ which are multiplied by the bare low-energy constants[6]

$$c_i = \delta_i \mu^{-\epsilon}\left(\frac{2}{\epsilon} - \gamma + \log 4\pi\right) + c_i^{\mathrm{r}}(\mu)\,. \tag{2.13}$$

Each constant contains a divergent part which absorbs the divergence that arises from loops with vertices from $\mathcal{L}_2$. The coefficient δ_i of the divergent term is unambiguously given within the EFT framework. The finite, scheme- and scale-dependent renormalized low-energy constants $c_i^{\mathrm{r}}(\mu)$ encode the different underlying theories, e.g. a strongly interacting gauge theory, like QCD, or some linear σ-model with a heavy Higgs boson. The renormalization group running of $c_i^{\mathrm{r}}(\mu)$ is again completely fixed within the EFT. In principle, these low-energy constants can be derived by performing a matching between effective and full theory at low energies. In practice, e.g. for a strongly interacting case, one has to determine them from experiment.

There is a well known procedure to construct all possible terms in the effective lagrangian at a given order in the low-energy expansion [12, 22, 23, 24, 25, 29]. First of all, it is useful to assign the following chiral dimensions to the different fields

$$U_p = \mathcal{O}(1)\,, \qquad G_p = \mathcal{O}(1)\,, \qquad D_\mu U_p = \mathcal{O}(p)\,, \qquad A_\mu^p = \mathcal{O}(1)\,. \tag{2.14}$$

These assignments determine the order of individual terms in the chiral lagrangian by power counting. In the present case, one has to take into account that there is a larger global chiral symmetry if we switch off all the gauge couplings in eq. (2.3). The lagrangian in eq. (2.2) is then invariant under the following $2N$ global chiral transformations of the N non-linear σ-fields U_p

$$U_p \to e^{i\xi_L^p} U_p e^{-i\xi_R^{p+1}}\,, \quad e^{i\xi_{L,R}^p} \in \mathrm{SU}(k)\,, \quad \xi_{L,R}^p = \xi_{L,R}^{p,a} T^a\,. \tag{2.15}$$

From comparison with eq. (2.5) one sees that at each point p the gauge group is just the diagonal of two adjacent chiral symmetry groups generated by ξ_L^p and ξ_R^p. We therefore require that under global chiral transformations the gauge fields transform as follows

$$A_\mu^p \to e^{i(\xi_L^p + \xi_R^p)/2} A_\mu^p e^{-i(\xi_L^p + \xi_R^p)/2}\,. \tag{2.16}$$

Half of the chiral symmetries are broken by the gauge interaction and every breaking is

[6]We will use dimensional regularization with $d = 4 - \epsilon$ which preserves chiral and gauge invariance. Furthermore contributions to the action from the integration measure $[\mathrm{d}U]$ in the path integral vanish in dimensional regularization.

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accompanied by a factor of the gauge coupling constant g. To keep track of the chiral symmetry breaking we introduce matrix-valued spurion fields,[7] that are external fields which for our purposes can be thought of being constant. Note that our spurion fields differ from those introduced in [12]. Their transformation properties are assigned as

$$q_L^p \to e^{i\xi_L^p} q_L^p e^{-i(\xi_L^p+\xi_R^p)/2} \qquad \text{and} \qquad q_R^p \to e^{i\xi_R^p} q_R^p e^{-i(\xi_L^p+\xi_R^p)/2} \,. \tag{2.17}$$

With their aid one can construct a lagrangian that is chirally invariant. The new covariant derivative is chosen such that it transforms correctly under chiral transformations

$$D_\mu U_p = \partial_\mu U_p - i q_L^p A_\mu^p q_L^{p\dagger} U_p + i U_p q_R^{p+1} A_\mu^{p+1} q_R^{p+1\dagger} \,. \tag{2.18}$$

At the end of the calculation, the spurions will be set to constant values $q_L, q_R \to \sqrt{g}\,\mathbb{1}$, therefore explicitly breaking the chiral symmetry. In order to have a consistent assignment for the chiral dimension of the covariant derivative, we choose the spurions to be of order $\mathcal{O}(p^{1/2})$. This means that we count the gauge coupling g as $\mathcal{O}(p)$, similarly to [29] which considered chiral perturbation theory with virtual photons. Contributions of higher order in the coupling constant g will therefore be automatically suppressed at low energies.

Making use of the spurions, the requirement of CP-invariance and translational invariance around the circle, the lowest order lagrangian $\mathcal{L}_2$, that is chirally invariant, contains only the two terms written in eq. (2.2), apart from gauge-fixing and ghost terms.

Since we will study in this paper only one-loop corrections to the masses of the gauge and Goldstone bosons, it will be sufficient to consider only the following four terms in $\mathcal{L}_4$ (the spurions have already been replaced by factors of $\sqrt{g}\,\mathbb{1}$)

$$\begin{aligned}\mathcal{L}_4 = {} & c_1 g^2 v^2 \sum_p \mathrm{Tr}\left[U_p^\dagger (D_\mu U_p)\, U_{p+1} (D^\mu U_{p+1})^\dagger\right] + \frac{c_2 g^2}{2} \sum_p \mathrm{Tr}\left[F_p^{\mu\nu} U_p F_{\mu\nu}^{p+1} U_p^\dagger\right] + \\ & + c_3 g^2 v^2 \sum_p \mathrm{Tr}\left[D_\mu U_p (D^\mu U_p)^\dagger\right] + c_4 g^2 \sum_p \mathrm{Tr}\left[F_{\mu\nu}^p F_p^{\mu\nu}\right] \cdots \,.\end{aligned} \tag{2.19}$$

For details see appendix B. A priori, there is another term of the form

$$\left|\mathrm{Tr}\left[q_R^1 q_L^{1\dagger} U_1 q_R^2 q_L^{2\dagger} U_2 \cdots U_N\right]\right|^2 \tag{2.20}$$

that is of $\mathcal{O}(p^{2N})$ and can potentially contribute. Since we are particulary interested in the large-N limit, we will choose $N \geq 4$, so that this term will not show up at $\mathcal{O}(p^4)$.

3. One-loop corrections to the gauge and Goldstone boson mass

In 5D gauge theory compactified on a circle the one-loop corrections to the Kaluza-Klein gauge-boson masses have been calculated by several authors [16, 17, 18, 19]. In the corresponding uncompactified theory, which is obtained in the limit of the compactification radius R going to infinity, the 5D gauge bosons are massless at the classical level ($p^\mu p_\mu = p_5^2$)

[7] During the preparation of this manuscript [30] appeared which deals with the topic of spurions in moose models in great detail.

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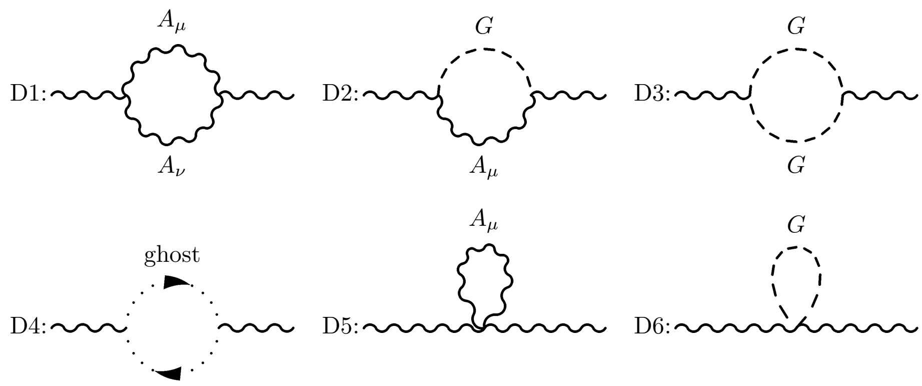

Figure 2: Diagrams contributing to the A_μ mass.

due to gauge and Lorentz invariance. Since this theory is non-renormalizable, radiative corrections can only be calculated by introducing a cutoff. If one introduces this cutoff in a way that respects gauge and Lorentz invariance, these symmetries ensure that the gauge boson remains massless even at the one-loop level. Since Lorentz invariance is violated in the infrared by the compactification, finite mass corrections (that vanish as $R \to \infty$) can be generated by radiative corrections. Here we want to investigate this issue in the context of deconstruction, which provides a manifestly gauge invariant regularization for the above mentioned theory. The corrections to the gauge boson masses at one-loop order are calculated for large but finite N and we show how to relate the results to the 5D theory.

We will find it advantageous to work in Feynman gauge ($\xi = 1$) where all fields have the same tree-level mass spectrum. Using the Feynman rules from $\mathcal{L}_2$ and the basis of wavefunctions from appendix A one can work out the contributions to $\Pi_{\mu\nu}$ from all six one-loop diagrams that contribute (see figure 2). For the calculation of the mass correction one is only interested in those terms that are proportional to $g_{\mu\nu}$.

$$\Pi_{\mu\nu}(q) = g_{\mu\nu}\Pi_1(q^2) - q_\mu q_\nu \Pi_2(q^2)\,, \qquad \lim_{q^2 \to m_n^2} \Pi_1(q^2) \simeq \delta m_n^2\,. \tag{3.1}$$

The last relation in eq. (3.1) is valid in the leading order approximation. Details on the calculation and the results for each individual diagram in figure 2 can be found in appendix C. Quite remarkably, using the formulae given there, it is possible to work out the sums in a closed form. However here we choose to expand in terms of N^{-1} to simplify the result. The contribution from all one-loop diagrams to δm_n^2 is (for $n > 0$)

$$\begin{aligned}\delta m_n^2\big|_{\text{loops}} = \frac{g^2 C_2(G)}{\pi^2}\Bigg\{ & \frac{m_n^2}{8}\left(1 + \frac{1}{12}\sin^2\frac{n\pi}{N}\right)\left[\frac{2}{\epsilon} - \gamma + \log 4\pi - \log\frac{g^2 v^2}{\mu^2}\right] + \\ & + \left[\frac{3}{4}\frac{g^2 v^2}{N^3}\zeta(3) - \frac{15}{4}\frac{g^2 v^2}{N^5}\zeta(5) + \mathcal{O}\left(\frac{1}{N^7}\right)\right] - \\ & - \left[\frac{7}{96} - \frac{3}{8N^3}\zeta(3) + \mathcal{O}\left(\frac{1}{N^5}\right)\right] m_n^2 + \\ & + \left[\frac{11}{96} + \mathcal{O}\left(\frac{1}{N^3}\right)\right] m_n^2 \sin^2\frac{n\pi}{N} + m_n^2\,\mathcal{O}\left(\sin^4\frac{n\pi}{N}\right)\Bigg\}\,. \end{aligned} \tag{3.2}$$

Apart from the corrections coming from loop diagrams, we also have to include contributions from the terms in $\mathcal{L}_4$. Due to their higher order in p they enter only at tree level. Their contribution to the gauge-boson mass originates from the four terms in eq. (2.19)

$$\delta m_n^2\big|_{\mathcal{L}_4} = g^2\mu^\epsilon m_n^2\Big((c_1+c_2+c_3+c_4) - 2(c_1+c_2)\sin^2\frac{n\pi}{N}\Big)\,. \tag{3.3}$$

The $1/\epsilon$ singularities appearing in eq. (3.2) are cancelled by the divergent part of the c_i (see eq. (2.13)). The renormalized parameters $c_i^{\rm r}$ are coupling constants of the non-linear σ-model. Since mass corrections proportional to m_n^2 and $m_n^2\sin^2 n\pi/N$ involve the constants $c_i^{\rm r}$ they can only be predicted if the $c_i^{\rm r}$ can be extracted by fits to additional experimental data.[8] If one makes a linear completion of this model these constants can be calculated in the context of a linear sigma model [20]. They will however depend on the details of the linearization such as the choice of the Higgs-potential.

The question remains how the 5D limit is obtained at one-loop order. To avoid the subtleties of the compactification, we will consider the limit of $N\to\infty$ which corresponds to $R\to\infty$ in the 5D theory. In this limit the 5D theory possesses full 5D Lorentz invariance. However since we are working in a latticized model we expect this symmetry to be violated at scales smaller than the lattice spacing a. When going to the uncompactified theory the fifth component of the momentum becomes a continuous variable (as before we have $2\pi R = Na$)

$$p_5^2 = \frac{n^2}{R^2}\,. \tag{3.4}$$

The corresponding relation for the mass m_n^2 reads

$$m_n^2 = 4g^2v^2\sin^2\frac{n\pi}{N} = 4g^2v^2\sin^2\frac{p_5 a}{2}\,. \tag{3.5}$$

We require that the violation of the Lorentz symmetry does not occur at distances larger than the lattice spacing a. Therefore we demand that

$$p_5^2(1+\mathcal{O}(p_5 a)) = \lim_{N\to\infty}(m_n^2+\delta m_n^2)\,. \tag{3.6}$$

Combining the one-loop corrections eqs. (3.2), (3.3) with the tree level expression we find that the mass in the limit $N\to\infty$ is given by

$$\lim_{N\to\infty}(m_n^2+\delta m_n^2) = 4g^2v^2\sin^2\frac{p_5 a}{2}\left[1+g^2\left(\sum_{i=1}^{4}c_i^{\rm r}(\mu) - \frac{C_2(G)}{8\pi^2}\left(\frac{7}{12}+\log\frac{g^2v^2}{\mu^2}\right)\right)\right]. \tag{3.7}$$

In order for eq. (3.6) to be fulfilled, the tree level relation for the lattice spacing in terms of parameters of the 4D EFT (eq. (2.6)) has to be modified by quantum corrections, as has been pointed out in [9]. The properly chosen lattice spacing at one-loop order, that fulfills eq. (3.6) is given by

$$a_{(1)}^2 = \frac{1}{v^2g^2}\left[1-g^2\left(\sum_{i=1}^{4}c_i^{\rm r}(\mu) - \frac{C_2(G)}{8\pi^2}\left(\frac{7}{12}+\log\frac{g^2v^2}{\mu^2}\right)\right)\right]. \tag{3.8}$$

[8] Because the constants $c_i^{\rm r}$ enter only in the linear combinations $c_1^{\rm r}+c_2^{\rm r}$ and $c_3^{\rm r}+c_4^{\rm r}$, to determine all of them, it is not sufficient to measure the mass splittings only, but one also has to consider other processes such as scattering amplitudes.

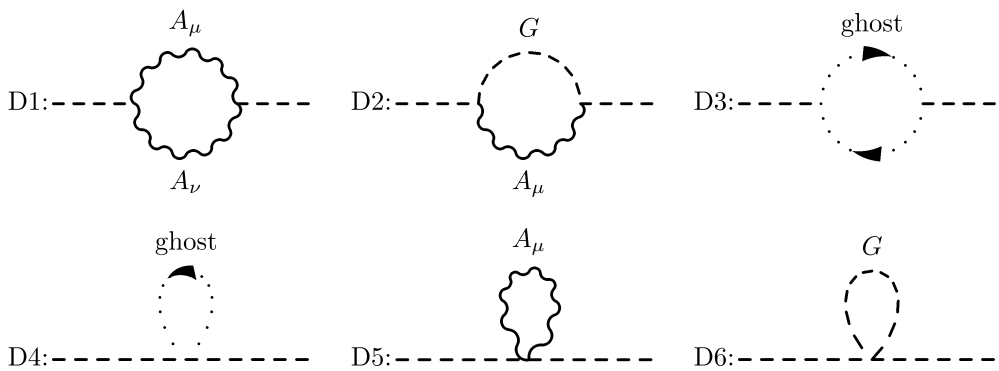

Figure 3: Diagrams contributing to the Goldstone-boson mass.

Note that since $m_n^2 + \delta m_n^2$ is μ-independent, $a_{(1)}$ is also independent of μ. With the appropriate definition of a the mass corrections are well defined in the limit $a \to 0$. Keeping the radius R fixed, one recovers the limit of a continuous and compactified fifth dimension. Introducing the 5D gauge coupling $g_5^2 = ag^2$ one finds that the mass corrections are given by

$$m_n^2 + \delta m_n^2 \to \frac{n^2}{R^2} + \frac{3}{4}\frac{g_5^2}{2\pi R}\frac{C_2(G)\zeta(3)}{4\pi^4 R^2}\,. \tag{3.9}$$

This is exactly the result obtained in the 5D calculations [17, 18]. Having worked in a manifestly gauge invariant framework and having maintained 5D Lorentz invariance in the $R \to \infty$ limit by the appropriate choice of the lattice spacing, we have obtained a finite result[9] as was discussed in the beginning of section 3.

We have not yet dealt with the zero mode. Since all contributions from $\mathcal{L}_4$ are proportional to m_n^2, which is zero in this case, they do not appear in the mass correction of the zero mode. The explicit calculation shows that the correction is in fact exactly zero, as all diagrams in figure 2 cancel each other. This is consistent with the 5D calculation, since in the limit of a continuous dimension the zero-mode mass has to vanish because of 5D gauge symmetry.

The situation for the mass of the Goldstone-boson is somewhat different from the case of the A_μ. Going to unitary gauge one can show that the only physical mode is the zero mode G_0. All other modes are eaten due to the breakdown of the gauge symmetry to the diagonal subgroup ($\mathrm{SU}(k)^N \to \mathrm{SU}(k)_{\mathrm{diag.}}$) and form the heavy gauge fields [12]. Similarly to the case of the A_μ zero-mode the mass corrections have to be finite due to the absence of terms in $\mathcal{L}_4$ that can absorb possible $1/\epsilon$-poles. However in this case the correction is not zero. The sum of diagrams from figure figure 3 becomes

$$\delta m_0^2 = \frac{g^2 C_2(G)}{\pi^2}\left[\frac{9}{4}\frac{g^2v^2}{N^3}\zeta(3) + \frac{45}{4}\frac{g^2v^2}{N^5}\zeta(5) + \mathcal{O}\left(\frac{1}{N^7}\right)\right]. \tag{3.10}$$

In the limit of going over to a continuous fifth dimension the shift is three times what we have found for the A_μ-mass (eq. (3.9)).

[9] This has already been pointed out, but not explicitly been shown in [18].

4. Conclusion

When doing calculations in 5D theories one is immediately confronted with the problem of non-renormalizability. Deconstruction is one way of introducing a gauge invariant UV-regulator in the theory, similar to conventional lattice regularization. Although the non-linear σ-model obtained by this procedure as a low energy EFT is non-renormalizable, one can apply the methods from chiral perturbation theory to obtain sensible and finite results at every order in the low energy expansion. We have illustrated how to apply these methods to the case of deconstruction and have given an explicit example by calculating the corrections to the masses of the gauge boson and the Goldstone boson in a non-abelian pure-gauge theory. In order to retain the correspondence between the 5D and the deconstructed theory at higher order, a sensible definition of the lattice spacing is necessary at a given order in perturbation theory to ensure 5D Lorentz invariance. With the appropriate definition the gauge and Goldstone boson masses become independent of the cutoff and the continuum limit can be taken. In this limit the one-loop mass corrections are finite due to 5D gauge and Lorentz invariance and agree with earlier 5D calculations [16, 17, 18, 19]. We have demonstrated that deconstruction, in the cases investigated in this paper, works even at the one-loop order and can potentially serve as a viable UV-completion for 5D theories.

Acknowledgments

We thank A. Falkowski and S. Pokorski for providing us with details on their related work on the linear σ-model [20] and for discussions. This research was supported in part by the National Science Foundation under the Grant No. PHY99-07949 and the Swiss National Science Foundation under Grant No. SNF20-68037.02. Z.K. and M.P. also thank the Kavli Institute for Theoretical Physics, UCSB for hospitality during the completion of this work.

A. Mode decomposition

We first introduce a basis of functions on the set of points $p = 1, \dots, N$ that fulfills the requirement of N-periodicity. The functions

$$g_n^+(p) = \sqrt{\frac{2}{N}} \cos \frac{2n\pi p}{N}\,, \qquad g_n^-(p) = \sqrt{\frac{2}{N}} \sin \frac{2n\pi p}{N}\,, \tag{A.1}$$
$$g_0^+(p) = \frac{1}{\sqrt{N}}\,, \qquad g_{N/2}^+(p) = \frac{1}{\sqrt{N}}(-1)^p\,, \qquad \text{with } n \in \left\{1, \dots, \frac{N}{2} - 1\right\},$$

form a complete set and serve as an orthonormal basis with respect to the scalar product $(\sigma, \sigma' \in \{+, -\})$

$$\sum_{p=1}^{N} g_n^\sigma(p) g_{n'}^{\sigma'}(p) = \delta_{nn'} \delta_{\sigma\sigma'}\,. \tag{A.2}$$

Expanded in this basis the gauge- and the ghost-field read

$$A_\mu^p = \sum_n \left(A_\mu^{+,n} g_n^+(p) + A_\mu^{-,n} g_n^-(p) \right), \qquad c_p = \sum_n \left(c_n^+ g_n^+(p) + c_n^- g_n^-(p) \right). \tag{A.3}$$

Since the Goldstone boson fields G_p are associated with the link between the points p and $p+1$ we choose a different Fourier basis

$$f_n^+(p) = \sqrt{\frac{2}{N}} \cos \frac{2n\pi(p+\frac{1}{2})}{N}\,, \qquad f_n^-(p) = -\sqrt{\frac{2}{N}} \sin \frac{2n\pi(p+\frac{1}{2})}{N}\,, \tag{A.4}$$
$$f_0^+(p) = \frac{1}{\sqrt{N}}\,, \qquad f_{N/2}^-(p) = \frac{1}{\sqrt{N}}(-1)^{p+1}\,, \quad \text{with } n \in \left\{1, \ldots, \frac{N}{2}-1\right\}.$$

The corresponding expansion for the Goldstone bosons reads

$$G_p = \sum_n \left(G_n^+ f_n^+(p) + G_n^- f_n^-(p) \right). \tag{A.5}$$

In this basis the free part of the lagrangian $\mathcal{L}_2$ becomes diagonal and we obtain the propagators

$$(D_{\mathrm{GB}}^{ab})_{nn'}^{\sigma\sigma'}(k) = \frac{i}{k^2 - \xi m_n^2 + i\epsilon} \delta^{\sigma\sigma'} \delta_{nn'} \delta^{ab}\,, \qquad (D_{\mathrm{ghost}}^{ab})_{nn'}^{\sigma\sigma'}(k) = \frac{i}{k^2 - \xi m_n^2 + i\epsilon} \delta^{\sigma\sigma'} \delta_{nn'} \delta^{ab}\,,$$
$$(D_{\mathrm{gauge}}^{\mu\nu,ab})_{nn'}^{\sigma\sigma'}(k) = -i\left[\frac{g^{\mu\nu}}{k^2 - m_n^2 + i\epsilon} - (1-\xi)\frac{k^\mu k^\nu}{(k^2 - \xi m_n^2 + i\epsilon)(k^2 - m_n^2 + i\epsilon)}\right] \delta^{\sigma\sigma'} \delta_{nn'} \delta^{ab}\,. \tag{A.6}$$

B. Lagrangian at $\mathcal{O}(p^4)$

For the purpose of this paper, where we study only the one-loop corrections to the masses of the gauge bosons and the Goldstone boson, it is not necessary to construct a complete list of operators at $\mathcal{O}(p^4)$ and then reduce it to a minimal set of independent operators, using algebraic identities and the equations of motion. In particular, the issue of preserving gauge invariance when using the equations of motion for the Goldstone and the gauge bosons is not so straightforward as in ordinary chiral perturbation theory in the case of local gauge symmetries, see [25]. Furthermore, since we do not consider other observables, like scattering amplitudes, we will not be able to extract for instance the divergent terms of individual low-energy constants c_i (eq. (2.13)). Therefore, we give below only some counter terms that are needed to render the masses of the gauge bosons finite. In particular, one finds the following four terms that give a contribution to the mass of A_μ, see also [30],

$$\begin{aligned}
&c_1 v^2 \sum_p \mathrm{Tr}\left[U_p^\dagger D_\mu U_p\, q_R^{p+1} q_L^{p+1\dagger}\, U_{p+1} \left(D^\mu U_{p+1}\right)^\dagger q_L^{p+1} q_R^{p+1\dagger}\right],\\
&\frac{c_2}{2} \sum_p \mathrm{Tr}\left[q_L^p F_p^{\mu\nu} q_L^{p\dagger}\, U_p\, q_R^{p+1} F_{\mu\nu}^{p+1} q_R^{p+1\dagger} U_p^\dagger\right],\\
&c_3 g^2 v^2 \sum_p \mathrm{Tr}\left[D_\mu U_p (D^\mu U_p)^\dagger\right],\\
&c_4 g^2 \sum_p \mathrm{Tr}\left[F_{\mu\nu}^p F_p^{\mu\nu}\right].
\end{aligned} \tag{B.1}$$

We have written the last two terms in a shorthand notation that comprises a set of terms that differ by various insertion of spurions. After setting the spurions to $\sqrt{g}\,\mathbb{1}$ they give the same contribution, which is proportional to the corresponding terms in the lowest order lagrangian $\mathcal{L}_2$. One can convince oneself that all terms in eq. (B.1) give contributions to the mass correction that are proportional to m_n^2 or $m_n^2 \sin^2(n\pi/N)$.[10] The same is true for the Goldstone boson where $m_n^2 = 0$. Therefore no counter term is needed at $\mathcal{O}(p^4)$ for the corrections to the mass of the Goldstone boson and the zero-mode of the gauge boson.

C. Diagrams contributing at one-loop order

In this section we give details on the calculation of all one-loop diagrams that contribute to the gauge boson self-energy (figure 2) in next-to-leading order. The first step is to obtain the Feynman rules. One expands the fields in terms of their Fourier modes eqs. (A.3), (A.5). Making use of trigonometric identities and using the orthonormality relations eq. (A.2) one can evaluate the sum over p and read off the vertex functions for the individual vertices. As an example we consider the case of the three-vertex of two gauge and one Goldstone boson field. For the special choice of parities given in figure 4 the vertex function becomes

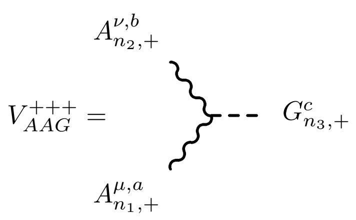

Figure 4: Two gauge, one Goldstone boson vertex.

$$V^{+++}_{AAG} = i\frac{g^2 v^2 N}{4} f^{abc}\Bigg[\sin\frac{(2n_1 - n_3)\pi}{N}\left(\tilde{\delta}_{n1-n2,n3} + \tilde{\delta}_{n1+n2,n3}\right) \\ - \sin\frac{(2n_1 + n_3)\pi}{N}\left(\tilde{\delta}_{n1-n2,-n3} + \tilde{\delta}_{n1+n2,-n3}\right)\Bigg]\eta_{n_1}\eta_{n_2}\eta_{n_3} g^{\mu\nu} \quad \text{(C.1)}$$

where

$$\tilde{\delta}_{n,n'} = \begin{cases} 1 & \text{if } (n-n') \bmod N = 0 \\ 0 & \text{else} \end{cases}, \qquad \eta_n = \begin{cases} \sqrt{1/N} & \text{if } n = 0, N/2 \\ \sqrt{2/N} & \text{else} \end{cases}. \quad \text{(C.2)}$$

The remaining vertex functions can be obtained in a similar way. With this information at hand, the expressions for all diagrams that contribute to the one-loop gauge boson self energy can be written down using the propagators from eq. (A.6). One is left with two sums over Kaluza-Klein labels of the propagating internal states, one of which can be evaluated by making use of the Kronecker delta. Because of translation invariance the diagrams are only non-vanishing when both external states have the same Kaluza-Klein label. One proceeds in the usual way by Feynman parameterizing the integrals and evaluating them in dimensional regularization. The results are given in table 1. For the calculation of the mass corrections we are interested in the on-shell limit of these expressions. Quite remarkably the remaining sums can be evaluated in a closed from with the aid of the following two

[10]To obtain higher powers in $\sin(n\pi/N)$ one would need to construct terms that involve products of fields at distant points p (e.g. $\mathrm{Tr}\left[q_L^p F^p_{\mu\nu} q_L^{p\dagger}\, U_p\, q_R^{p+1} q_L^{p+1\dagger}\, U_{p+1}\, q_R^{p+2} F^{\mu\nu}_{p+2} q_R^{p+2\dagger}\, U^\dagger_{p+1}\, q_L^{p+1} q_R^{p+1\dagger}\, U^\dagger_p\right]$ for contributions of the form $m_n^2 \sin^4(n\pi/N)$), but these terms are of higher power in momentum due to the spurion fields.

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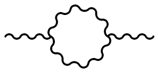
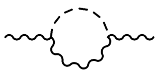
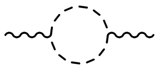


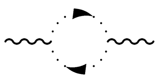
$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \int_0^1 \mathrm{d}x \Big[\big(-3(3-\epsilon)\Gamma(-1+\epsilon/2)\Delta^{1-\epsilon/2} + \big((2-x)^2+(1+x)^2\big)q^2\Gamma(\epsilon/2)\Delta^{-\epsilon/2}\big)g^{\mu\nu} + \big((2-\epsilon)(1-2x)^2-(1+x)(2-x)\big)\Gamma(\epsilon/2)\Delta^{-\epsilon/2}q^\mu q^\nu \Big]$$



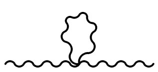
$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \sin^2\frac{(n-n')\pi}{N} \int_0^1 \mathrm{d}x \big(-2(4g^2v^2)\Gamma(\epsilon/2)\Delta^{-\epsilon/2}\big)g^{\mu\nu}$$



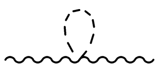
$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \cos^2\frac{n'\pi}{N} \int_0^1 \mathrm{d}x \times \Big[-2\Gamma(-1+\epsilon/2)\Delta^{1-\epsilon/2}g^{\mu\nu} + (1-2x)^2\Gamma(\epsilon/2)\Delta^{-\epsilon/2}q^\mu q^\nu \Big]$$



$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \int_0^1 \mathrm{d}x \Big[\Gamma(-1+\epsilon/2)\Delta^{1-\epsilon/2}g^{\mu\nu} + 2x(1-x)\Gamma(\epsilon/2)\Delta^{-\epsilon/2}q^\mu q^\nu \Big]$$



$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \int_0^1 \mathrm{d}x \big[(3-\epsilon)(4-\epsilon)\Gamma(-1+\epsilon/2)\Delta^{1-\epsilon/2} - 2(3-\epsilon)\big((1-x)^2q^2 - m^2_{n+n'}\big)\Gamma(\epsilon/2)\Delta^{-\epsilon/2}\big]g^{\mu\nu}$$



$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \cos\frac{2n'\pi}{N} \int_0^1 \mathrm{d}x \big[(4-\epsilon)\Gamma(-1+\epsilon/2)\Delta^{1-\epsilon/2} - 2\big((1-x)^2q^2 - m^2_{n+n'}\big)\Gamma(\epsilon/2)\Delta^{-\epsilon/2}\big]g^{\mu\nu}$$

Table 1: Corrections to the gauge boson propagator ($\Delta = (1-x)m_n^2 + xm_{n+n'} - x(1-x)q^2$ and n' labels the external states).

relations (here ψ_0 denotes the Digamma-function).[11]

$$\sum_{n=-(N/2-1)}^{N/2} \sin^{2m}\Big(\frac{n\pi}{N}\Big) = \frac{N}{2^{2m}}\binom{2m}{m}, \tag{C.3}$$

$$\sum_{n=-(N/2-1)}^{N/2} \sin^{2m}\Big(\frac{n\pi}{N}\Big)\log\Big[\sin^2\frac{n\pi}{N}\Big] = \frac{1}{2^{2m-1}}\Bigg[\sum_{k=1}^{m}(-1)^k\binom{2m}{m-k}\Big(\psi_0\Big(\frac{k}{N}\Big)+\psi_0\Big(1+\frac{k}{N}\Big)\Big) - \binom{2m}{m}(\gamma+N\log 2)\Bigg]. \tag{C.4}$$

The mass correction eq. (3.2) follows from expanding the rather lengthly result in the case of large N. In the special case of the zero-mode with $m^2_{n'} = 0$ the correction is found to be vanishing.

[11] Eq. (C.4) can be derived by the use of Gauss's Digamma theorem [31].

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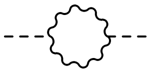
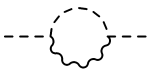
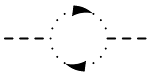


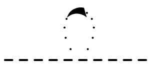
$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \cos^2\frac{n\pi}{N}\int_0^1 \mathrm{d}x\Big[4(4-\epsilon)m_n^2\Gamma(\epsilon/2)\tilde{\Delta}^{-\epsilon/2}\Big]$$



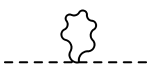
$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \cos^2\frac{n\pi}{N}\int_0^1 \mathrm{d}x \times$$
$$\Big[(4-\epsilon)\Gamma(-1+\epsilon/2)\tilde{\Delta}^{1-\epsilon/2} - 2q^2(1+x)^2\Gamma(\epsilon/2)\tilde{\Delta}^{-\epsilon/2}\Big]$$



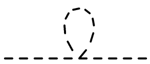
$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \cos^2\frac{n\pi}{N}\int_0^1 \mathrm{d}x\Big[-2m_n^2\Gamma(\epsilon/2)\tilde{\Delta}^{-\epsilon/2}\Big]$$



$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \sin^2\frac{n\pi}{N}\int_0^1 \mathrm{d}x\Big[-\frac{4}{3}\Gamma(-1+\epsilon/2)m_n^{2-\epsilon}\Big]$$



$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} \cos\frac{2n\pi}{N}\int_0^1 \mathrm{d}x\Big[-2(4-\epsilon)\Gamma(-1+\epsilon/2)m_n^{2-\epsilon}\Big]$$



$$= \frac{ig^2\mu^\epsilon C_2(G)}{2N(4\pi)^{2-\epsilon/2}} \sum_{n=-\left(\frac{N}{2}-1\right)}^{\frac{N}{2}} (4g^2v^2)^{-1}\int_0^1 \mathrm{d}x \times$$
$$\Big[\frac{1}{3}(4-\epsilon)\Gamma(-2+\epsilon/2)m_n^{4-\epsilon} - \frac{2}{3}q^2\Gamma(-1+\epsilon/2)m_n^{2-\epsilon}\Big]$$

Table 2: Corrections to the Goldstone boson propagator ($\tilde{\Delta} = m_n^2 - x(1-x)q^2$).

Next we want to do the corresponding calculation for the Goldstone boson. Here the situation is somewhat easier since we only have to deal with the zero-mode, being the only physical degree of freedom. Proceeding as before the contributing diagrams from figure 3 give the corrections listed in table 2. After evaluating the sums and expanding in N^{-1} the mass correction is the one given in eq. (3.10).

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